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A note on constrained shape memory alloys upon thermal cycling

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Abstract In this paper, we present a systematical study on the constrained shape memory alloys upon thermal cycling. The focuses are the transformation temperature interval and hysteresis. Closed-form expressions are obtained for biased actuators and thin films atop an elastic substrate.

Introduction

Shape memory alloy (SMA) is considered as one of the most promising smart materials for micro actuators, largely due to its large recovery force over a long actuation distance. Naturally, there are three types of SMAs based actuators ([1]) (Fig. 1),

– Figure 1a illustrates a one-way actuator. The SMA is elongated initially at a low temperature. Upon heating, due to the recovery of the SMA, element *P* is pulled to the left;
– Figure 1b shows a biased actuator, which is capable of moving the element *P* back or forth by heating or cooling the SMA element;
– Figure 1c is a two-way actuator, which has two SMA elements. *P* can move back and forth by heating and cooling the two SMA elements alternatively.

Among them, the second type (biased actuator) is more of practical interest. Due to the effect of the spring, the transformation temperatures of the SMA can be alter dramatically as observed in many experiments (e.g. [2]). Utilizing this phenomenon, [3] proposed to extend the so-called temperature memory effect into a wider temperature range. From the engineering application point view, it is important to know the temperature range and hysteresis in actuation. As such, a quick estimation in the pre-design stage is particularly helpful. It appears to be necessary to systematically investigate the relationship between the transformation temperatures of a SMA and the constraint and other conditions, e.g., the stiffness of the elastic spring, and geometry of each component, etc.

To the best knowledge of the authors, perhaps the first person who systematically studied biased actuators in a configuration as shown in Fig. 1b is [4]. However, according to this numerical simulation, upon heating, the stress in SMAs increases continuously. It sounds odd, since the thermal expansion of SMAs should reduce the internal stress. Recently, [5] qualitatively discussed the transformation temperature interval and hysteresis in constrained SMAs upon thermal cycling. The purpose of this paper is to quantitatively study the biased actuator (as shown in Fig. 2) and SMA thin films atop an elastic substrate. The latter is popular in micro-electromechanical systems (MEMS) (e.g., [6]) and practically is one particular type of biased actuator. The transformation temperature interval and hysteresis upon thermal cycling are of our current interest.

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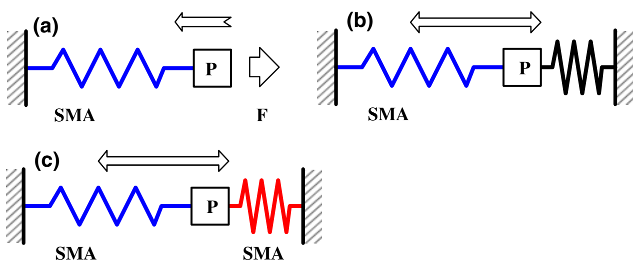

Fig. 1 Basic types of SMA actuators. (**a**) One-way actuator, (**b**) biased actuator, and (**c**) two-way actuator [1]

Biased actuator

As shown in Fig. 2, consider a piece of SMA that is pre-stretched and then connected to an elastic spring. Assume that only the SMA is under thermal cycling, while the temperature of the spring is a constant. At a particular moment (for instance, upon cooling), in which the development of the martensite fraction is $d\xi_M$, one has

$$k = -\frac{d\sigma}{\frac{L_M}{L}\left(\frac{d\sigma}{E_M} + \alpha_M dT\right) + \frac{L_A}{L}\left(\frac{d\sigma}{E_A} + \alpha_A dT\right) + \varepsilon^p d\xi_M}\frac{A}{L} \tag{1}$$

where k is the stiffness of the spring at this particular moment, ε^p is the phase transformation strain in $d\xi_M$, α and E are the coefficient of thermal expansion and Young's modulus of the SMA, A and L are the cross-sectional area and length of the SMA. Here, subscripts M and A stand for martensite and austenite, respectively. $d\sigma$ and dT are the changes in stress and temperature in the SMA, respectively.
As an approximation, one has

$$\begin{cases} \frac{L_M}{L} = \xi_M \\ \frac{L_A}{L} = 1 - \xi_M \end{cases} \tag{2}$$

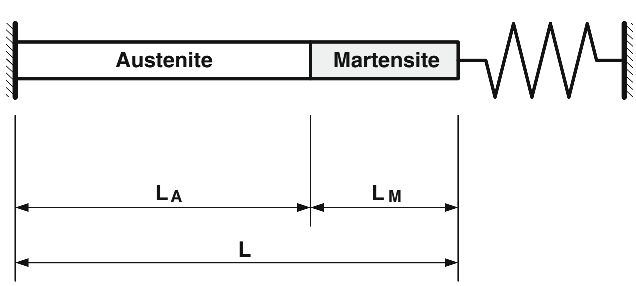

Fig. 2 Schematic illustration of a SMA linked to an elastic spring

where ξ_M, the martensite fraction, is a function of T and σ. Substituting Eq. 2 into Eq. 1 yields

$$\begin{aligned} \varepsilon^p d\xi_M = &-\left[\frac{A}{Lk} + \frac{\xi_M}{E_M} + \frac{(1-\xi_M)}{E_A}\right]d\sigma \\ &- [\xi_M\alpha_M + (1-\xi_M)\alpha_A]dT \end{aligned} \tag{3}$$

Practically, it is rather difficult to obtain a precise solution for ξ_M. In the literature, a few functions [4, 7, 8] have been proposed for it. However, as pointed out by [9], the exact transformation progress in a SMA not only varies from one material to another, but also depends on the exact applied stress state. Here, we take the simplest expression for ξ_M, i.e., we assume that ξ_M is a linear function of T and σ,

$$\begin{aligned} \xi_M = &\overline{A}(\sigma - \hat{\sigma}) + \overline{B}\left(T - \hat{T}\right) \\ &(\xi_M \in [0,1],\ \overline{A} > 0 \text{ and } \overline{B} < 0) \end{aligned} \tag{4}$$

where $\overline{A}, \overline{B}, \hat{\sigma}$ and $\hat{T}$ are constants. Hence,

$$d\xi_M = \overline{A}d\sigma + \overline{B}dT \tag{5}$$

In order to obtain explicit expressions in the following discussion, we further assume that $E_A \approx E_M = E$ and $\alpha_A \approx \alpha_M = \alpha$. Thus, Eq. 3 can be simplified as

$$\varepsilon^p\left(\overline{A}d\sigma + \overline{B}dT\right) = -\left(\frac{A}{Lk} + \frac{1}{E}\right)d\sigma - \alpha dT \tag{6}$$

Re-writing Eq. 6 results in

$$\frac{d\sigma}{dT} = -\frac{\varepsilon^p\overline{B} + \alpha}{\frac{A}{Lk} + \frac{1}{E} + \varepsilon^p\overline{A}} \tag{7}$$

Equation (7) is always valid even without the phase transformation in SMAs. In such a case, in Eq. 7, $\varepsilon^p = 0$.

Refer to Fig. 3. If k is more or less a constant, i.e., the spring is linear elastic, upon cooling from a high temperature (point o), where the SMA is austenite, the slope of $\overline{oa}$ is

$$\left(\frac{d\sigma}{dT}\right)_{\overline{oa}} = -\frac{\alpha}{\frac{A}{Lk} + \frac{1}{E}} \tag{8}$$

Upon further cooling, at point a, the martensitic transformation starts. One may take $\varepsilon^p = \varepsilon^p_{max}$, where ε^p_{max} is the maximum phase transformation strain of the SMA, which can be determined experimentally (e.g., [8]), so that

$$\left(\frac{d\sigma}{dT}\right)_{\overline{ab}} = -\frac{\varepsilon^p_{\max}\overline{B} + \alpha}{\frac{A}{Lk} + \frac{1}{E} + \varepsilon^p_{\max}\overline{A}} \quad (9)$$

From point b to c, there is no more martensitic transformation, since the SMA is already 100% martensite. As such,

$$\left(\frac{d\sigma}{dT}\right)_{\overline{bc}} = -\frac{\alpha}{\frac{A}{Lk} + \frac{1}{E}} \quad (10)$$

Upon heating, from point c to d, it is dominated by elastic unloading. The corresponding slope is

$$\left(\frac{d\sigma}{dT}\right)_{\overline{cd}} = -\frac{\alpha}{\frac{A}{Lk} + \frac{1}{E}} \quad (11)$$

However, in practice, upon cooling from point b to c, the stress in SMA increases continuously, which normally causes a small amount of stress-induced martensite reorientation. Hence, the real slope of $\overline{bc}$ is smaller, i.e., $\overline{bc}$ and $\overline{cd}$ are not overlapped.

Upon further heating, from point d to e, the reverse transformation starts. The slope can be expressed as

$$\left(\frac{d\sigma}{dT}\right)_{\overline{de}} = -\frac{\varepsilon^p_{\max}\overline{B} + \alpha}{\frac{A}{Lk} + \frac{1}{E} + \varepsilon^p_{\max}\overline{A}} \quad (12)$$

Heating further, at point e, the SMA is 100% austenite. From point e too, the deformation is elastic, and the slope is

$$\left(\frac{d\sigma}{dT}\right)_{\overline{eo}} = -\frac{\alpha}{\frac{A}{Lk} + \frac{1}{E}} \quad (13)$$

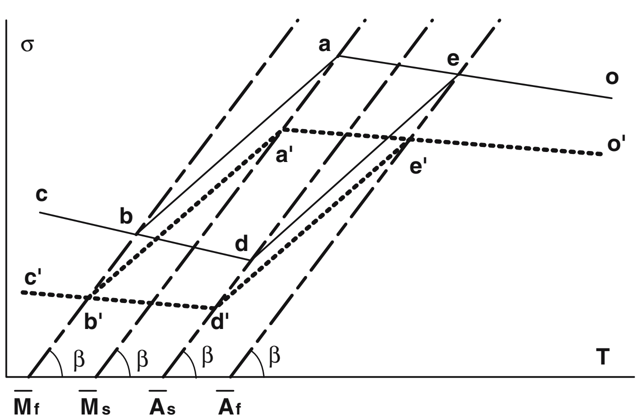

Fig. 3 T vs. σ relationship of a SMA upon thermal cycling. Solid line and dotted line correspond to the responses of two different springs

It is clear that apart from the thermomechanical properties and geometry of SMA, these slopes also depend upon k, the stiffness of the spring. If another thermal cyclic test is carried out using another spring (a different k), two curves as illustrated in Fig. 3 by solid line and dotted line are obtained. It is well known that the four transformation temperatures, namely, martensite finish temperature (M_f), martensite start temperature (M_s), austenite start temperature (A_s) and austenite finish temperature (A_f), are approximately linear functions of the applied external stress [10]. Theoretically, a and a' belong to a straight line (Fig. 3), which represents the relationship between M_s and σ. So are b and b', and other critical points. For simplicity, we assume that the slopes of these lines are the same ($\tan(\beta)$).

The transformation temperature interval (ΔT) indicates the temperature range between the start and finish of the phase transformation (e.g., $\Delta T = \overline{M}_s - \overline{M}_f$ in Fig. 3 for the thermally induced martensitic transformation). In the case of constrained SMAs,

$$\Delta T = \frac{\tan(\beta)}{\tan(\beta) + \frac{\varepsilon^p_{\max}\overline{B}+\alpha}{\frac{A}{Lk}+\frac{1}{E}+\varepsilon^p_{\max}\overline{A}}}\left(\overline{M}_s - \overline{M}_f\right) \quad (14)$$

According to Eq. 14,

- Given a SMA, ΔT depends on the geometry (A/L) of the SMA and the stiffness of the spring (k).
- The maximum ΔT is

$$\Delta T_{\max} = \frac{\tan(\beta)}{\tan(\beta) + \frac{\varepsilon^p_{\max}\overline{B}+\alpha}{\frac{1}{E}+\varepsilon^p_{\max}\overline{A}}}\left(\overline{M}_s - \overline{M}_f\right) \quad (15)$$

which is obtained when $k = \infty$, i.e., the length of SMA is fixed during thermal cycling.

Another important issue in SMAs is hysteresis [11]. The temperature hysteresis (ΔH) in a thermal cycle can be expressed as (Fig. 3)

$$\Delta H = T_e - T_a = \frac{\tan(\beta)}{\tan(\beta) + \frac{\alpha}{\frac{A}{Lk}+\frac{1}{E}}}\left(\overline{A}_f - \overline{M}_s\right) \quad (16)$$

Equation (16) reveals that

- Given a SMA, ΔH depends on the geometry (A/L) of the SMA and the stiffness of the spring (k).
- The minimum ΔH is

$$\Delta H_{\min} = \frac{\tan(\beta)}{\tan(\beta) + \alpha E}\left(\overline{A}_f - \overline{M}_s\right) \quad (17)$$

which is obtained when $k = \infty$, i.e., the length of SMA is fixed during thermal cycling.
With ΔT and ΔH, one can determine the evolution of stress and strain in the SMA during thermal cycling.

SMA thin film atop an elastic substrate

Thin films deposited atop a substrate are popular in MEMS applications. SMA thin films, in particular, NiTi and NiTiCu thin films, have been used in a variety of devices [12, 13].

A common technique to produce SMA thin films atop a substrate is sputter deposition. If the deposition is carried out at a high temperature (normally above 400 °C), the resultant thin films are crystallized and with the shape memory effect. Otherwise, post-annealing is required for crystallization. Instead of annealing the whole wafer, a laser can be used to anneal only a small local area and leave the remaining part amorphous [14]. Theoretically, at a high sputtering/annealing temperature, the internal stress in SMA thin films is very small.

Apart from using a temperature controlled atomic force microscope to determine the transformation temperatures of SMAs in a small area at micron and/or submicron scales [15], a more standard approach is to measure the curvature (κ) of SMA thin films atop an elastic substrate upon thermal cycling. Provided that the SMA film is reasonably thin and the substrate is in a disk-shape, given the κ vs. T relationship, one can determine the evolution of stress (σ) and strain (ε) in a SMA thin film upon thermal cycling [16], i.e., (refer to Fig. 4)

$$\begin{cases} \sigma = -\dfrac{\kappa E_s h^2}{6t(1-\nu_s)} \\ \varepsilon = \frac{2}{3}\kappa h + (\alpha_s - \alpha)T \end{cases} \tag{18}$$

where E_s, ν_s and α_s are the Young's modulus, coefficient of thermal expansion and Poisson's ratio of the substrate, t and h are the thickness of the SMA film and substrate. Subsequently, one can get the σ vs. T relationship of the SMA thin film, which is more or less the same in shape as that in Fig. 3. In addition,

$$\varepsilon = \frac{\sigma(1-\nu)}{E} + \xi_M \varepsilon^{p'}_{max} \tag{19}$$

where ν is the Poisson's ratio of the SMA film and $\varepsilon^{p'}_{max}$ is the maximum transformation strain in the biaxial tensile or compressive stress state. From Eqs. 18 and 19, one has

$$\varepsilon = -\frac{4t(1-\nu_s)\sigma}{E_s h} + (\alpha_s - \alpha)T = \frac{\sigma(1-\nu)}{E} + \xi_M \varepsilon^{p'}_{max} \tag{20}$$

After differentiating Eq. 20 and substituting Eq. 5 into it, one has

$$\frac{d\sigma}{dT} = \frac{\alpha_s - \alpha - \varepsilon^{p'}_{max}\overline{B}}{\frac{(1-\nu)}{E} + \frac{4t(1-\nu_s)}{E_s h} + \varepsilon^{p'}_{max}\overline{A}} \tag{21}$$

It is the slope of $\overline{ab}$ and $\overline{de}$ in Fig. 3. If one deletes the phase transformation terms in Eq. 21, the slope of $\overline{oa}$ and $\overline{cd}$ is

$$\left(\frac{d\sigma}{dT}\right)_{\overline{oa}} = \left(\frac{d\sigma}{dT}\right)_{\overline{cd}} = \frac{\alpha_s - \alpha}{\frac{(1-\nu)}{E} + \frac{4t(1-\nu_s)}{E_s h}} \tag{22}$$

Subsequently, the transformation temperature interval

$$\Delta T = \frac{\tan(\beta)}{\tan(\beta) - \frac{\alpha_s - \alpha - \varepsilon^{p'}_{max}\overline{B}}{\frac{(1-\nu)}{E} + \frac{4t(1-\nu_s)}{E_s h} + \varepsilon^{p'}_{max}\overline{A}}}\left(\overline{M}_s - \overline{M}_f\right) \tag{23}$$

and the temperature hysteresis

$$\Delta H = \frac{\tan(\beta)}{\tan(\beta) - \frac{\alpha_s - \alpha}{\frac{(1-\nu)}{E} + \frac{4t(1-\nu_s)}{E_s h}}}\left(\overline{A}_f - \overline{M}_s\right) \tag{24}$$

In the case that a SMA thin film atop a silicon substrate, ($\alpha_s < \alpha$, so that the SMA thin film is in tension), when $t/h \to 0$, the maximum transformation temperature interval is

$$\Delta T_{max} = \frac{\tan(\beta)}{\tan(\beta) - \frac{\alpha_s - \alpha - \varepsilon^{p'}_{max}\overline{B}}{\frac{(1-\nu)}{E} + \varepsilon^{p'}_{max}\overline{A}}}\left(\overline{M}_s - \overline{M}_f\right) \tag{25}$$

and the minimum temperature hysteresis is

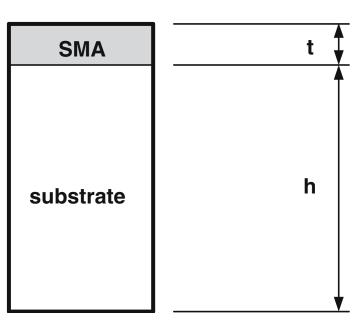

Fig. 4 Illustration of a SMA thin film atop a substrate

$$\Delta H_{\min} = \frac{\tan(\beta)}{\tan(\beta) - \frac{\alpha_s - \alpha}{1-v}E}\left(\overline{A}_f - \overline{M}_s\right) \tag{26}$$

If $\alpha_s = 0$

$$\Delta T_{\max} = \frac{\tan(\beta)}{\tan(\beta) + \frac{\alpha + \varepsilon_{\max}^{p'}\overline{B}}{\frac{(1-v)}{E} + \varepsilon_{\max}^{p'}\overline{A}}}\left(\overline{M}_s - \overline{M}_f\right) \tag{27}$$

and

$$\Delta H_{\min} = \frac{\tan(\beta)}{\tan(\beta) + \frac{\alpha}{1-v}E}\left(\overline{A}_f - \overline{M}_s\right) \tag{28}$$

Conclusions

In this paper, we present a quantitative study of constrained SMAs upon thermal cycling. Two cases are investigated, namely, SMA wire based biased actuator and SMA thin film atop an elastic substrate. Closed-form solutions for the transformation temperature interval and temperature hysteresis of both cases are obtained. We find that given a particular SMA, in a biased actuator, with the increase of L/A and/or k, ΔT increases, while ΔH decreases. The maximum ΔT and minimum ΔH correspond to the case that the SMA is fixed during thermal cycling. In the case that a SMA thin film is deposited atop a silicon wafer, ΔT increases with the decrease of t/h, while the trend for ΔH is in opposite. With ΔT and ΔH, one can determine the variations in stress and strain of a SMA upon thermal cycling.

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